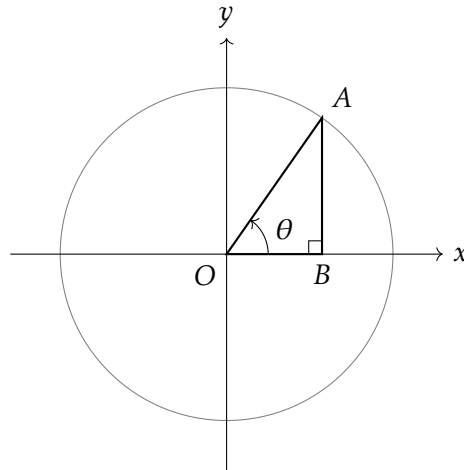


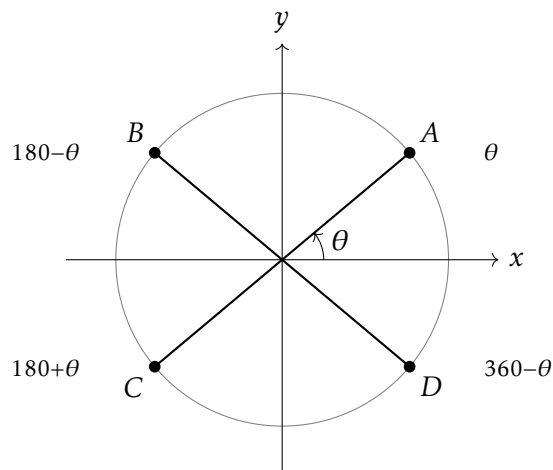
The Unit Circle

Things you should already know

- $\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$, $\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$, $\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$
- SOHCAHTOA works for angles between 0° and 90°
- Right-angled triangle trigonometry



The Four Quadrants



Example

In which quadrants is each trig function positive?

Example

Without a calculator, determine the sign of each:

(a) $\sin 130^\circ$

(b) $\cos 210^\circ$

(c) $\tan 315^\circ$

Textbook Exercises: SPSC: 4.10 — Exercise 1A, 1B.

Dr Frost: Trigonometric Graphs slides — unit circle and CAST diagram.

Corbett Maths: Trig Graphs practice questions (Video 362).

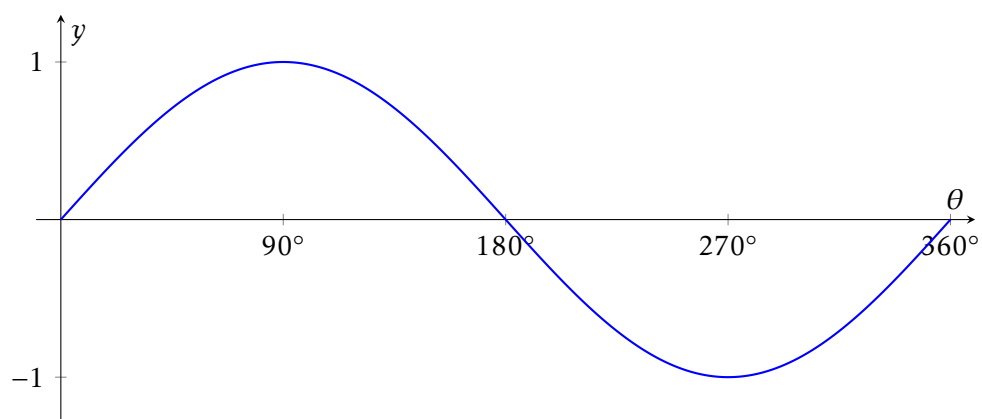
Maths Genie: Trigonometric Graphs worksheet.

Graphs of the Trigonometric Functions

Example

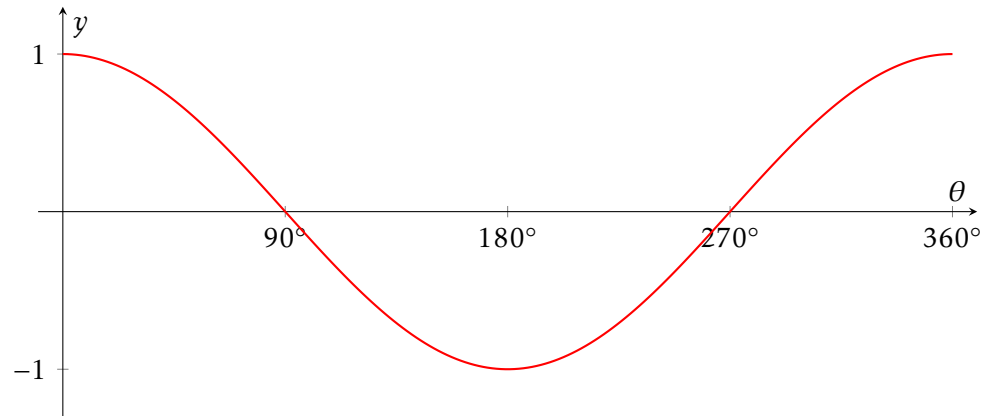
Using the unit circle, fill in the table for $y = \sin \theta$:

θ	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
$\sin \theta$													

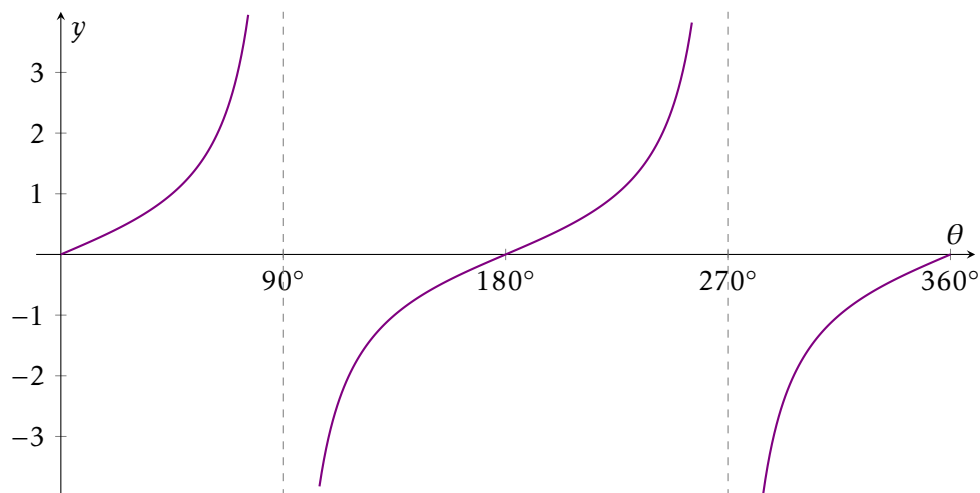
The graph of $y = \sin \theta$ 

Key properties of $y = \sin \theta$:

The graph of $y = \cos \theta$



Key properties of $y = \cos \theta$:

The graph of $y = \tan \theta$ 

Key properties of $y = \tan \theta$:

Example

You are told that $\sin 24^\circ = 0.407$. Without a calculator, write down the values of:

(a) $\sin 156^\circ$

(b) $\sin 204^\circ$

(c) $\sin 336^\circ$

Textbook Exercises: SPSC: 4.10 — Exercise 1C.

Dr Frost: Trigonometric Graphs slides — sketching and reading values from graphs.

Corbett Maths: Trig Graphs practice questions.

Maths Genie: Trigonometric Graphs worksheet (symmetry questions).

Solving Trigonometric Equations

Example

Using a sketch of $y = \sin \theta$ for $0^\circ \leq \theta \leq 360^\circ$, how many solutions does $\sin \theta = 0.5$ have? What about $\sin \theta = -0.3$?

Method

Fact — To solve a trigonometric equation over $0^\circ \leq \theta \leq 360^\circ$:

1. Use your calculator to find the **first solution** (the “principal value”).
2. Use the **symmetry of the graph** to find the **second solution**.
3. Check both solutions are in the required range.

Example

Solve $\sin \theta = 0.6$ for $0^\circ \leq \theta \leq 360^\circ$. Give answers to 1 d.p.

Example

Solve $\cos \theta = -0.4$ for $0^\circ \leq \theta \leq 360^\circ$. Give answers to 1 d.p.

Example

Solve $\tan \theta = -2$ for $0^\circ \leq \theta \leq 360^\circ$. Give answers to 1 d.p.

Example

Solve $\sin \theta = -0.5$ for $0^\circ \leq \theta \leq 360^\circ$.

Extension: Equations of the form $\sin k\theta = c$

Fact — For equations like $\sin 2\theta = c$ or $\cos 3\theta = c$:

1. Let $u = k\theta$ and solve $\sin u = c$ in the range $0^\circ \leq u \leq 360k^\circ$
2. Divide all solutions by k to get θ

There are usually $2k$ solutions in any 360° range.

Example

Solve $\cos 2\theta = 0.5$ for $0^\circ \leq \theta \leq 360^\circ$.

Textbook Exercises: SPSC: 4.10 — Exercise 2A, 2B.

Dr Frost: Trigonometric Graphs slides — solving trig equations section.

Corbett Maths: Solving Trigonometric Equations practice questions (Video 363).

Maths Genie: Solving Trigonometric Equations worksheet.

Maths4Everyone: Trigonometric Equations booklet.